Statistics

Lecture 11

Feb 19-8:47 AM

SG12

Some Review

There are 12 Females and 18 males.

Randomly take one person

P(Select one female) $= \frac{12}{30} = \boxed{\frac{2}{5}} = \boxed{.4}$

odds in favor of Selecting one female

Females : # Females

12 : 18 $\boxed{2:3}$

12 $\boxed{\div}$ 18 $\boxed{\text{Math}}$ $\boxed{1:\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$ $\frac{2}{3}$

odds against Selecting one female $\boxed{3:2}$

Oct 13-10:44 AM

odds in favor of LA Rams win the Super Bowl this year are 3:17.

1) odds against $\boxed{17:3}$

2) $P(\text{Win}) = \frac{3}{3+17} = \boxed{\frac{3}{20}}$

3) $P(\overline{\text{Win}}) = \frac{17}{3+17} = \boxed{\frac{17}{20}}$

$P(\text{Win}) + P(\overline{\text{Win}}) = \frac{3}{20} + \frac{17}{20} = 1$

Oct 13-10:49 AM

Multiplication Rule

Keyword AND

Multiple Action Event

$P(A \text{ and } B)$

A happens,

B happens

Case I: Independent Events

One outcome does not change the prob. of next outcome

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B)}$$

Oct 13-10:52 AM

What is the prob. that two newborn babies are both girls?

$P(\text{Girl and Girl}) = \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}} = \boxed{.25}$

Roll a fair die twice, what is the prob. of getting two 6's?

$P(\text{Two 6's}) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

Oct 13-10:56 AM

Draw 2 Cards from a full deck of playing Cards, 52 Cards, 26 Red, 12 face, 4 aces.
with replacement

$P(\underbrace{\text{Two aces}}_{\text{Rare}}) = \frac{4}{52} \cdot \frac{4}{52} = \boxed{\frac{1}{169}} \approx \boxed{.006} \leq .05$

4 $\boxed{\div}$ 52 $\boxed{\times}$ 4 $\boxed{\div}$ 52 $\boxed{\text{Math}}$ $\boxed{\text{1:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$

$\boxed{\text{Math}}$ $\boxed{\text{2:}\blacktriangleright\text{Dec}}$ $\boxed{\text{Enter}}$

$P(\text{Two Faces}) = \frac{12}{52} \cdot \frac{12}{52} = \boxed{\frac{9}{169}} \approx .053 > .05$

Not rare

Oct 13-11:01 AM

$P(A) = .4$, $P(B) = .5$

A & B are independent events

1) $P(\bar{A}) = 1 - P(A)$
$= \boxed{.6}$

2) $P(A \text{ and } B) = P(A) \cdot P(B)$
$= (.4)(.5) = \boxed{.2}$

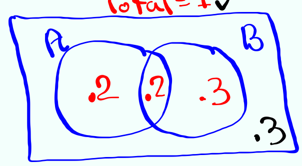
3) $P(\overline{A \text{ and } B})$
$= 1 - P(A \text{ and } B)$
$= 1 - .2 = \boxed{.8}$

4) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .4 + .5 - .2 = \boxed{.7}$

5) $P(\overline{A \text{ or } B})$
$= 1 - P(A \text{ or } B)$
$= 1 - .7 = \boxed{.3}$

6) Construct Venn Diagram

Total = 1 ✓



Oct 13-11:09 AM

$P(A) = .7$

$P(B) = .4$

A & B are independent events.

1) $P(\bar{A}) = 1 - P(A) = \boxed{.3}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.6}$

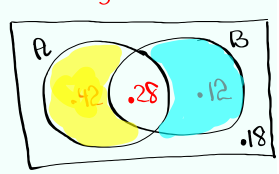
3) $P(A \text{ and } B) = P(A) \cdot P(B)$
$= (.7)(.4) = \boxed{.28}$

4) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .7 + .4 - .28$
$= \boxed{.82}$

$.7 - .28 = .42$

$.4 - .28 = .12$

5) Construct Venn Diagram



Total = 1

6) $P(A \text{ only or } B \text{ only}) = .42 + .12 = \boxed{.54}$

7) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = \boxed{.18}$
$= 1 - P(A \text{ or } B) = 1 - .82 = \boxed{.18}$

8) $P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = \boxed{.72}$
$= 1 - P(A \text{ and } B) = 1 - .28 = \boxed{.72}$

Oct 13-11:19 AM

Prob. of landing tails on a coin is $\frac{1}{3}$.

Not a fair coin

$P(T) = \frac{1}{3}$

$P(H) = \frac{2}{3}$

toss this twice

$P(\text{Two tails}) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

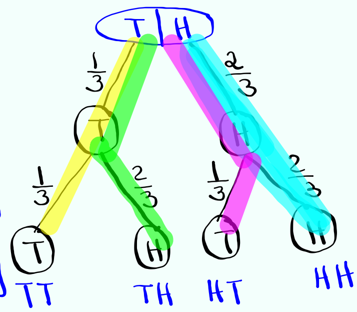
$P(\text{Two heads}) = \frac{2}{3} \cdot \frac{2}{3} = \boxed{\frac{4}{9}}$

Tree Diagram

$P(TT) = \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{1}{9}}$

$P(HH) = \frac{2}{3} \cdot \frac{2}{3} = \boxed{\frac{4}{9}}$

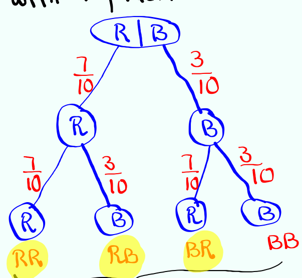
$P(1H\,1T) = 2 \cdot \frac{1}{3} \cdot \frac{2}{3} = \boxed{\frac{4}{9}}$



Oct 13-11:31 AM

A box has 7 Red & 3 Blue balls.

Take two balls with replacement.



Complete list of all possible outcomes

Sample Space

$P(RR) = \frac{7}{10} \cdot \frac{7}{10} = \boxed{\frac{49}{100}}$

$P(BB) = \frac{3}{10} \cdot \frac{3}{10} = \boxed{\frac{9}{100}}$

$P(1R \,\&\, 1B) = 2 \cdot \frac{7}{10} \cdot \frac{3}{10} = \boxed{\frac{42}{100}}$

$P(\text{at least 1 Red ball}) = 1 - P(\text{No Red}) = 1 - P(BB)$

Total Prob

$= 1 - \frac{9}{100} = \boxed{\frac{91}{100}}$

SG 12 ✓

Oct 13-11:39 AM

Multiplication Rule

Dependent events — one outcome impacts the Prob. of next outcome.

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B|A)}$$

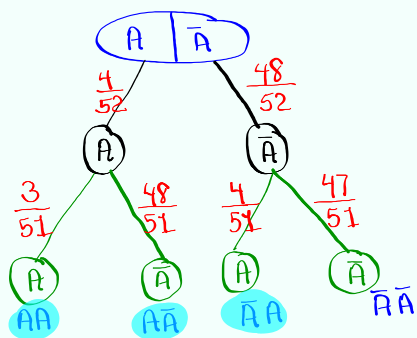
Given

A full deck has 52 Cards, 26 Red, 12 Face, and 4 aces. Draw 2 cards No replacement

$P(2 \text{ aces}) = \frac{4}{52} \cdot \frac{3}{51} = \boxed{\frac{1}{221}}$

$P(2 \text{ Faces}) = \frac{12}{52} \cdot \frac{11}{51} = \boxed{\frac{11}{221}}$

Oct 13-11:49 AM



$P(\text{at least 1 ace}) = 1 - P(\text{No aces})$

$= 1 - P(\bar{A}\bar{A})$

$= 1 - \frac{48}{52} \cdot \frac{47}{51}$

$= \boxed{\frac{33}{221}} \approx .149$

Draw 3 Cards, no replacement,

$P(\text{at least 1 ace}) = 1 - P(\text{no Aces}) = 1 - \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50}$

Total Prob.

$= \boxed{\frac{1201}{5525}}$

Oct 13-11:56 AM

4 Females, 6 Males, Select 4 people

$$P(\text{All are Females}) = \frac{4}{10}\cdot\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{1}{7} = \boxed{\frac{1}{210}}$$

$$P(\text{All are Males}) = \frac{6}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7} = \boxed{\frac{1}{14}}$$

$$P(\text{are all Same Sex}) = P(\text{FFFF or MMMM})$$
$$= \frac{1}{210} + \frac{1}{14} = \boxed{\frac{8}{105}}$$

$$P(\text{are not all Same Sex}) = 1 - P(\text{are Same Sex})$$
$$= 1 - \frac{8}{105} = \boxed{\frac{97}{105}}$$

$$P(\text{at least 1 Female}) = 1 - P(\text{No Females})$$
$$= 1 - P(\text{all males})$$
$$= 1 - \frac{1}{14} = \boxed{\frac{13}{14}}$$

FFFF

Some F & Some M

MMMM

Oct 13-12:06 PM

$$P(A \text{ and } B) = P(A)\cdot P(B\,|\,A)$$

Given

Divide by P(A)

$$\frac{P(A \text{ and } B)}{P(A)} = P(B\,|\,A)$$

$$\boxed{P(B\,|\,A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Conditional Prob.

Oct 13-12:17 PM

$P(A) = .6$

$P(B) = .5$

$P(A \text{ and } B) = .4$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.4}{.6} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

$$= \boxed{.667}$$

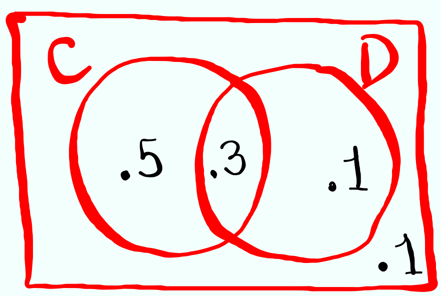
$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.4}{.5} = \frac{4}{5} = \boxed{.8}$$

Oct 13-12:20 PM

$P(\text{Coffee}) = .8$

$P(\text{Donut}) = .4$

$P(\text{Coffee and Donut}) = .3$



Total=1

$$P(\text{Donut} \mid \text{Coffee}) = \frac{P(C \text{ and } D)}{P(C)} = \frac{.3}{.8} = \frac{3}{8} = \boxed{.375}$$

$$P(\text{Coffee} \mid \text{Donut}) = \frac{P(C \text{ and } D)}{P(D)} = \frac{.3}{.4} = \frac{3}{4} = \boxed{.75}$$

Oct 13-12:24 PM

$P(A) = .5$

$P(B) = .8$

$P(A|B) = .6$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$.6 = \frac{P(A \text{ and } B)}{.8}$$

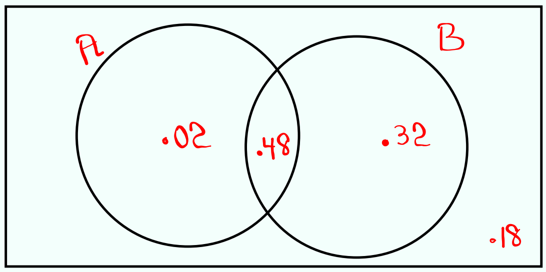
$P(A \text{ and } B)$

Cross-Multiply

$P(A \text{ and } B) = (.8)(.6)$

$= \boxed{.48}$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.48}{.5} = \boxed{.96}$$



Total = 1

Oct 13-12:30 PM